Int. J. Heat Mass Transfer. Vol. 30, No. 9, pp. 1839–1846, 1987
Printed in Great Britain

0017-9310/87 $3.00+0.00

A hydrodynamic model developed for the condensate flowing over a sinusoidal fluted tube

HÜSEYIN GÖKÇE and CANAN ÖZGEN†
Middle East Technical University, Ankara, Turkey

(*Received* 16 *July* 1986)

Abstract—Fluted tubes which were designed by Gregorig considering the use of surface tension forces are commonly used in evaporators and condensers for desalination purposes. In this study the surface profile of a fluted tube, which cannot be described by any conventional coordinate system is obtained numerically by using an intrinsic-coordinate system. Navier–Stokes equations are solved after some simplifications for the condensate flowing over the sinusoidal fluted profile to find the volumetric flow rate. The results are found to be in good agreement with the experimental findings.

1. INTRODUCTION

HIGH HEAT transfer performance condensers are required in the development of new energy sources. Among the many new methods for increasing condensation heat transfer this paper deals with the enhancement of the heat transfer coefficient using modified heat transfer surfaces (e.g. fluted tubes).

The fluted tube profile was first proposed by Gregorig in 1954 [1]. The tube walls are given sinusoidal shapes on the plane perpendicular to the tube axis having several flutes around the circumference. A flute is defined with two 'crests' and one 'groove' the specifications of which are further defined with the radii of curvature of the crests and the groove, the distance between the two crests, the depth of the flute and angle of the flute. In these tubes, the curvature difference of the sinusoidal profile of the tube wall on the plane perpendicular to the tube axis, develops pressure differences in the condensate film due to surface tension forces. In vertical fluted tube condensers, the condensate forming on the outer surface of the tube drains from crests to the groove of a flute as a result of surface tension forces, leaving bare areas over the crests. The condensate flows in the grooves in the direction of gravity. This film thinning over the crests considerably reduces the resistance to heat transfer and thus, increases the average condensing film coefficient (Fig. 1) compared to smooth tubes [2–7].

Although there are many experimental studies on fluted tubes as given above, there are only a few theoretical studies [8–11]. This is mainly due to the complex nature of the surface shape of the sinusoidally designed fluted tube where the analytical solution of Navier–Stokes equations is practically impossible for the condensate.

In this study, the intrinsic coordinate system is utilized to find the surface profile of the condensate falling down the outer surface of a fluted tube. A model is adapted for the solution of the Navier–Stokes equation in order to find the volumetric flow rate of the condensate.

2. DETERMINATION OF THE SURFACE PROFILE OF A FLUTED TUBE

Yorkshire Imperial Metals Limited was the supplier of the fluted tube tested. According to their information, the tube walls were sinusoidally shaped. Therefore, studies are directed to obtain a hydrodynamic model for sinusoidal fluted profiles. The close study of the fluted tube with a Vernier microscope by Özgen [12] has shown that both the outer and the inner surfaces do not fit a sinusoidal profile. However, the centreline passing through the half-thickness of the fluted wall fits a sinusoidal shape.

This centreline which cannot be described by any conventional coordinate system, is obtained by using an intrinsic coordinate system which is shown in Fig. 2. The centreline curvature, K, of a fluted surface is given by Wang [13] as

$$K = A + B\cos(\lambda s) \tag{1}$$

where A is the mean curvature, B the reciprocal of the amplitude of the corrugation, λ the frequency of the corrugation and s the arc length in radians measured from a fixed point on the curve. The Frenet–Serret formulas give the coordinates (x, y) of this centreline

$$\frac{d\phi}{ds} = K, \quad \frac{dx}{ds} = \cos\phi, \quad \frac{dy}{ds} = \sin\phi. \tag{2}$$

Since the mean curvature is not zero ($A \neq 0$), all

† Author to whom correspondence should be addressed.

NOMENCLATURE

a	radius of curvature of water from surface [mm]
A	constant in equation (1)
B	reciprocal of the amplitude of corrugation [mm^{-1}]
C_1, C_2	constants in equation (33)
g	gravitational acceleration, 9.80 m s^{-2}
h	film thickness [mm]
H	maximum film thickness [mm]
L_1	length of thin film section [mm]
N	number of flutes
$\mathbf{N}$	unit vector in Fig. 2
R_1, R_2	radius of curvature in Fig. 1
R	position vector of point X in intrinsic coordinates
R_w	R at the tube wall
R_h	R at the condensate surface
s	arc length
T	tangent vector in Fig. 2
$\Delta\dot{V}$	volumetric flow rate [ml min^{-1}]
V_z, V_v, V_θ	velocity vectors [m s^{-1}]
w	quantity in equation (16)
x, y, z	Cartesian coordinates
Y_0	quantity given in equation (21)
Δx	tube wall thickness [mm]
Δz	interval of tube length [m].

Greek symbols

α	flute angle [deg]
β	B/A in equation (3)
γ	angle measured from tube centre [rad]
γ_2	angle at which surface and condensate profiles are tangent to each other
ε	$R_{h_{avg}}/R_{w_{avg}}$
ξ	$r/R_{w_{avg}}$
κ	curvature
ϕ	angle between T- and X-axis
ρ	density of water [kg m^{-3}]
λ	frequency of corrugation
θ, θ_1	angles facing condensate film surface [deg]
θ_2	$\pi/2-\alpha$ [rad]
μ	viscosity [Pa s].

Superscript

$\tilde{}$	dimensionless quantity.

Subscripts

avg	average quantity.

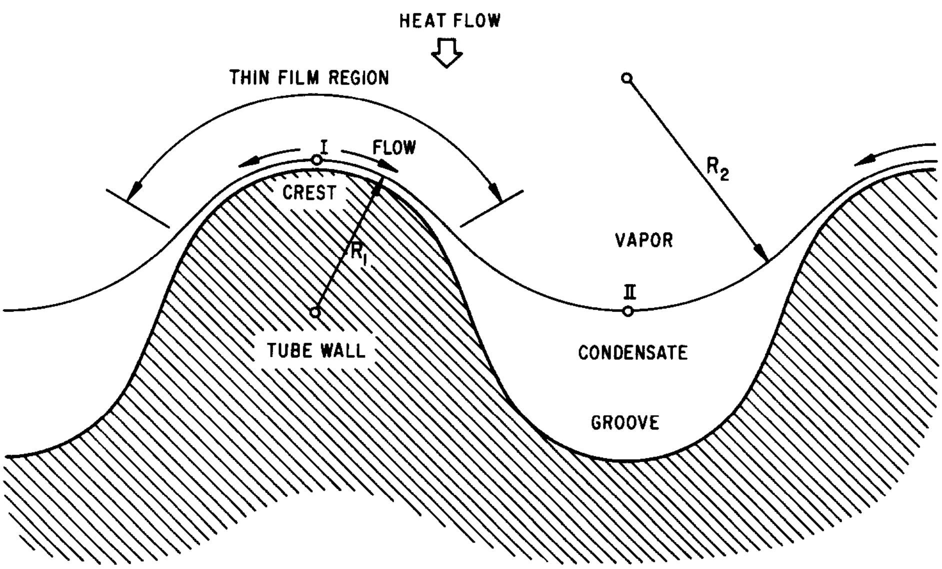


FIG. 1. Fluted tube—principle of operation.

lengths can be normalized by $1/A$. The dimensionless quantities are

$$\tilde{K}=\frac{K}{A},\quad \tilde{s}=sA,\quad \tilde{\lambda}=\frac{\lambda}{A},\quad \tilde{x}=xA,\quad \tilde{y}=yA.$$

Then

$$\frac{d\phi}{d\tilde{s}}=1+\beta\cos(\tilde{\lambda}\tilde{s}) \tag{3}$$

where

$$\beta=B/A$$

and

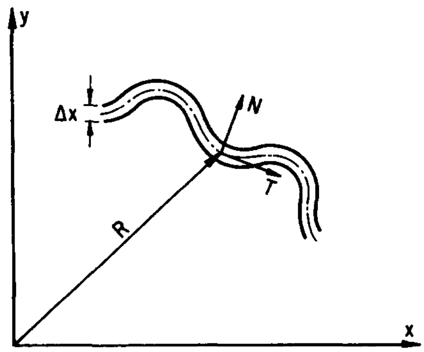


FIG. 2. The intrinsic coordinate system.

$$\frac{d\tilde{x}}{d\tilde{s}} = \cos\phi \tag{4}$$

$$\frac{d\tilde{y}}{d\tilde{s}} = \sin\phi. \tag{5}$$

Integration of equation (3) gives

$$\int_0^{\phi} d\phi = \int_0^{s} [1+\beta\cos(\tilde{\lambda}\tilde{s})]\,d\tilde{s} \tag{6}$$

$$\phi = \tilde{s} + \frac{\beta}{\tilde{\lambda}}\sin(\tilde{\lambda}\tilde{s}). \tag{7}$$

Substituting equation (7) into equations (4) and (5) yields

$$\frac{d\tilde{x}}{d\tilde{s}} = \cos\left(\tilde{s} + \frac{\beta}{\tilde{\lambda}}\sin(\tilde{\lambda}\tilde{s})\right) \tag{8}$$

$$\frac{d\tilde{y}}{d\tilde{s}} = \sin\left(\tilde{s} + \frac{\beta}{\tilde{\lambda}}\sin(\tilde{\lambda}\tilde{s})\right). \tag{9}$$

The primary interest is the centreline configuration of a corrugated tube with N-fold symmetry. Letting the total perimeter length be $\tilde{s}^t$; from equation (3) it can be intuitively concluded that

$$\tilde{\lambda}\tilde{s}^t = 2\pi N. \tag{10}$$

The quantities $\tilde{x}$ and $\tilde{y}$ are periodic for a closed tube in $(0, \tilde{s}^t)$, then equation (8) or (9) gives

$$\tilde{s}^t = 2\pi. \tag{11}$$

Equations (10) and (11) can be combined to give

$$\tilde{\lambda} = N. \tag{12}$$

Equations (8) and (9) can be written as

$$\frac{d\tilde{x}}{d\tilde{s}} = \cos\left(\tilde{s} + \frac{\beta}{\tilde{\lambda}}\sin(N\tilde{s})\right) \tag{13}$$

$$\frac{d\tilde{y}}{d\tilde{s}} = \sin\left(\tilde{s} + \frac{\beta}{\tilde{\lambda}}\sin(N\tilde{s})\right). \tag{14}$$

Numerical integration of equations (13) and (14) is carried out using second-order Euler integration and the dimensionless arc length, $\tilde{s}$, is increased as the integration proceeds (Fig. 3). A plot of the centreline configuration is given in Fig. 4. Table 1 lists the specifications of the tested tube.

A value for $\tilde{s}$ is chosen as $\tilde{s}_2$ on the centreline. This dimensionless arc length corresponds to a specific angle γ_2 (Fig. 3). Knowing these values and the tube wall thickness, the angle θ_2 where the condensate profile is tangent to the outer tube wall surface can be calculated. In these calculations an assumption is made by taking the slope of the centreline at $\tilde{s}_2$ to be equal to the slope of the tube surface at the same γ_2. The error involved in this approximation is small if the wall thickness is small. This is also discussed by Wang [13]. When β-values are small (equations (13) and (14)), the same approximation is also valid because small β-values do not yield large slopes.

Since condensation takes place on the outer surface of a fluted tube, it is necessary to relate the outer surface to the centreline profile.

Referring to Fig. 3, the outer surface profile can be calculated by adding the thickness w in the direction of R at any angle γ. Then the magnitude of the position vector R_w for the outer surface is

$$R_w = R + w \tag{15}$$

and w is given by

$$w = \frac{\Delta x/2}{\sin(90-\gamma-\phi)}. \tag{16}$$

Angles γ and ϕ can be defined in terms of x and y

$$90-\gamma = \tan^{-1}\left(\frac{y}{x}\right) \tag{17}$$

$$\phi = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) \tag{18}$$

where ϕ indicates the local angle of inclination which can be calculated numerically.

3. DETERMINATION OF CONDENSATE SURFACE PROFILE

Somer and Özgen [7] showed that the surface of the condensate film over the grooves fits a circle of radius 'a' changing with the dimensions of the flute and the condensate flow rate (Fig. 5). The circle of the condensate surface is tangent to the tube surface at an angle θ_2 since the surface profile is not sinusoidal, a straight line connects the curvature at the crests and the grooves giving a constant value for θ_2. However, a sinusoidal profile yields an angle θ_2 changing with unit normal vector N along the arc length, s. Geometrical identities yield

$$\theta_2 = \phi. \tag{19}$$

The radius of the circle for the condensate surface results in

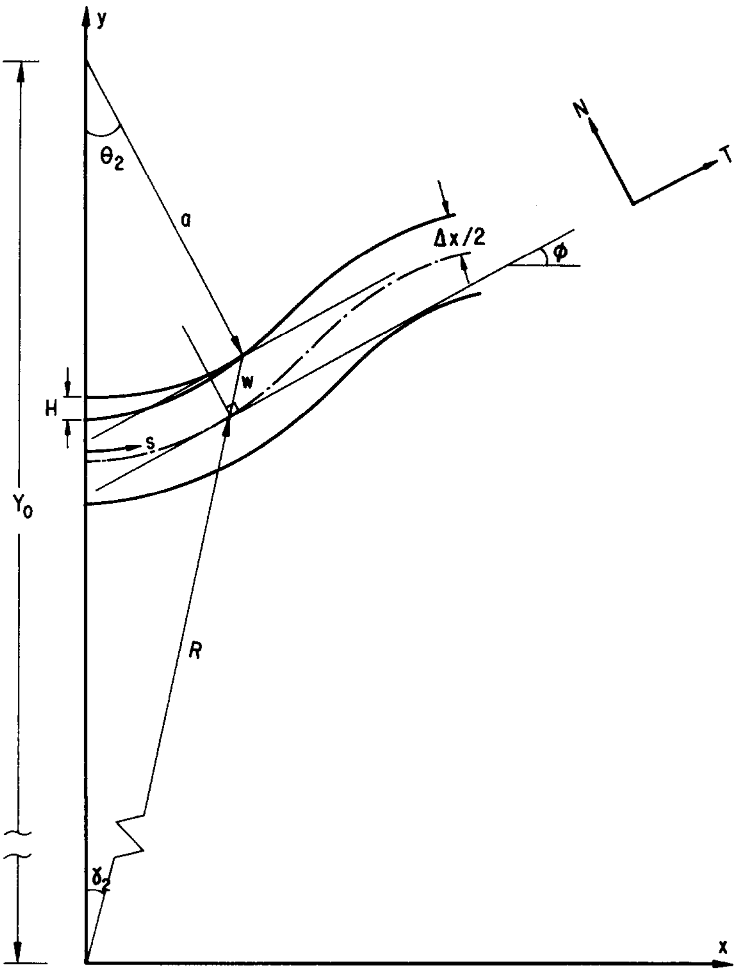


FIG. 3. Geometrical diagram used for the calculation of the outer surface and condensate profile.

$$a = \frac{R_w \sin\gamma}{\sin\theta_2}. \quad (20)$$

Hence, the length, Y_0, and the maximum film thickness, H, can be found as

$$Y_0 = a\cos\theta_2 + R_w\cos\gamma \quad (21)$$

$$H = Y_0 - a - R_w\Big|_{\gamma=0}. \quad (22)$$

Equation (22) enables one to calculate R_h at $\gamma = 0$ as

$$R_h\Big|_{\gamma=0} = R_w\Big|_{\gamma=0} + H. \quad (23)$$

For any other γ, R_h can be calculated from

$$R_h = \frac{R_h^2 - a^2 + Y_0^2}{2Y_0\cos\gamma}. \quad (24)$$

The condensate surface profile, R_h, is calculated by using the Wegstein convergence method. The value of R_h calculated in each step in the s-direction is taken as an initial estimate for the next step. The results of calculations are shown in Fig. 6.

4. DETERMINATION OF THE CONDENSATE FLOW RATE

The condensate formed over the whole surface of a fluted tube drains through the grooves in the downward direction and the flow is laminar [12]. In order to solve the condensate flow rate Navier–Stokes equations [14] in polar coordinates are used. The z-component of the equations has the form

$$\rho\left(\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right] + \rho g_z \quad (25)$$

and can be simplified with the assumptions given below.

(1) Flow in the r- and θ-directions inside the groove are negligible; $V_r = V_\theta = 0$.

(2) The condensate film is very thin and the resultant of all the forces acting on this film in the z-

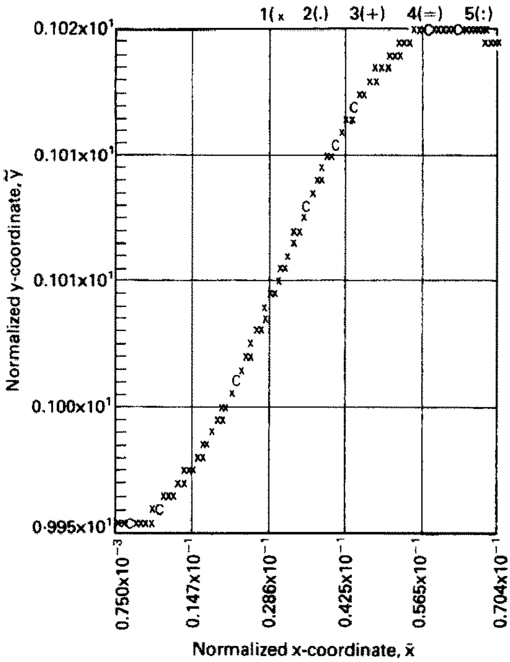


FIG. 4. The plot of the centreline configuration of the sinusoidal fluted tube tested.

Table 1. Specifications of the fluted tube tested

Diameter [mm]	50.8
Number of flutes, N	50
Plain thickness, x [mm]	0.812
Outside perimeter, P_o [m]	0.1972
Inside perimeter, P_i [m]	0.1922
Depth, d [mm]	1.08
Pitch, p [mm]	3.19
Length, L [m]	1.80
Radius of groove curvature, a [mm]	0.4
Radius of crest curvature, b [mm]	0.8
Flute angle, α [deg]	45
Total flute length, S_T [mm]	3.945

direction is zero. Considering that the pressure of the vapour in the chamber is also uniform; $\partial P/\partial z = 0$.

(3) The physical properties are constant (i.e. small variations in density, viscosity and surface tension due to slight variations in temperature and pressure can be neglected.

(4) The flow is steady.

Equation (25) then reduces to

$$\rho V_z \frac{\partial V_z}{\partial z} = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right] + \rho g_z. \quad (26)$$

Equation (26) can be further simplified by taking advantage of the equation of continuity, which yields

$$\frac{\partial V_z}{\partial z} = 0 \quad (27)$$

and hence

$$\frac{\partial^2 V_z}{\partial z^2} = 0. \quad (28)$$

Equation (26) becomes

$$0 = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}\right] + \frac{\rho g_z}{\mu}. \quad (29)$$

The exact solution of equation (29) for the given system is not possible and an approximate solution as described below is used.

If the flow area of the condensate in the groove area is divided into small sections in the θ-direction, then each $\Delta\theta$ interval can be considered as a thin film falling down a cylindrical surface having a constant film thickness over that interval. Note that γ is equivalent to θ in the conventional representation of polar coordinates (Fig. 3). Then the film thickness over the interval $\Delta\gamma$ is the arithmetic average of the film thickness at the start and at the end of the division. For this small $\Delta\gamma$ interval, equation (29) reduces to

$$0 = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}V_z}{\mathrm{d}r}\right) + \frac{\rho g_z}{\mu}. \quad (30)$$

The problem can now be solved with the following boundary conditions:

$$\text{(i) at } r = R_{h_{avg}}, \quad \frac{\mathrm{d}V_z}{\mathrm{d}r} = 0 \quad (31)$$

$$\text{(ii) at } r = R_{w_{avg}}, \quad V_z = 0 \quad (32)$$

where $R_{h_{avg}}$ is defined by

$$R_{h_{avg}} = R_{w_{avg}} + h_{avg}$$

where h_{avg} is the average film thickness over the interval, $\Delta\gamma$, and $R_{w_{avg}}$ is the average distance of the tube surface from the tube centre. Equation (30) can be integrated to give

$$V_z = -\frac{\rho g}{4\mu}r^2 + C_1 \ln r + C_2. \quad (33)$$

Application of the boundary conditions, equations (31) and (32), results in

$$C_1 = \frac{\rho g}{2\mu}(R_{h_{avg}})^2$$

$$C_2 = \frac{\rho g}{4\mu}R^2_{w_{avg}} - C_1 \ln R_{w_{avg}}.$$

Equation (33) can be written as

$$V_z = \frac{\rho g_z}{4\mu}R^2_{w_{avg}}\left[1 - \left(\frac{1}{R_{w_{avg}}}\right)^2 + 2\left(\frac{R_{h_{avg}}}{R_{w_{avg}}}\right)^2 \ln\frac{r}{R_{w_{avg}}}\right]. \quad (34)$$

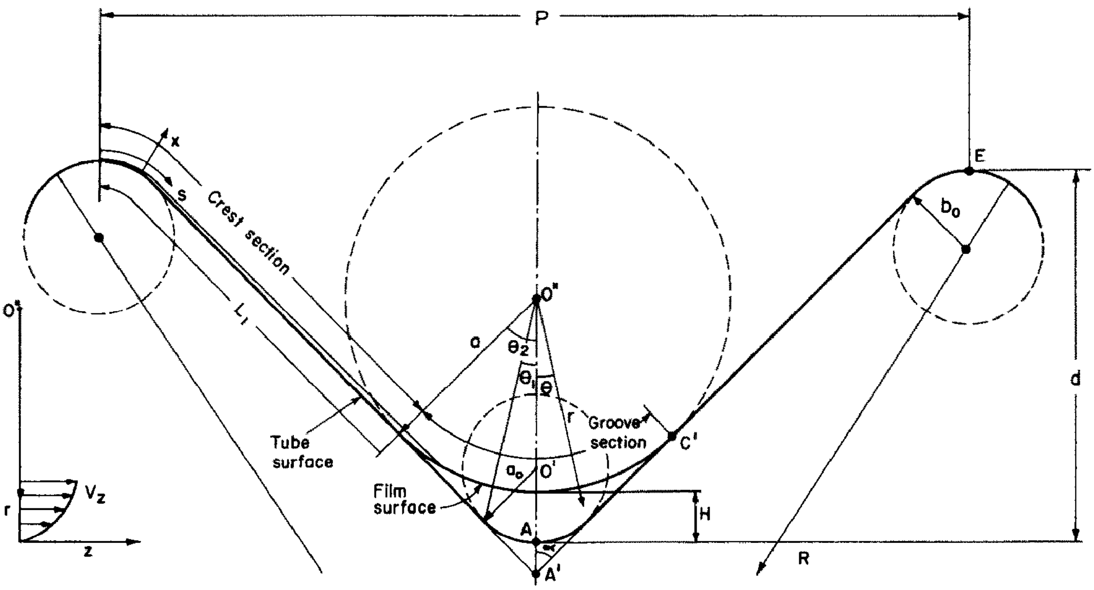
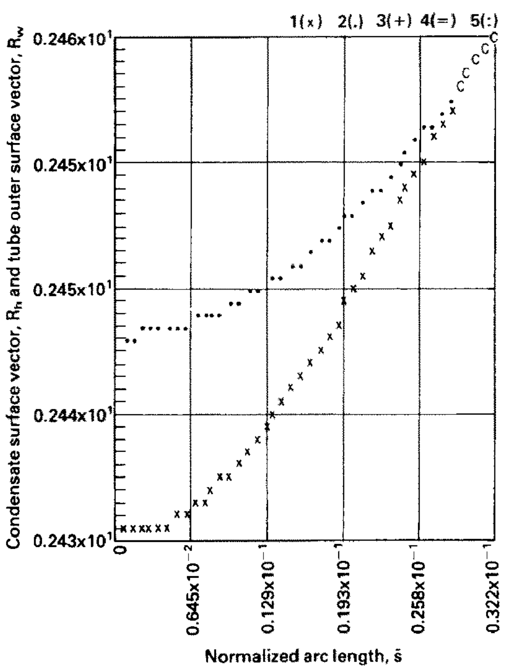


FIG. 5. Condensate profile over a flute.



A = 0.13730x10^1 mm
H = 0.14505 mm
V = 0.18946 ml/min

FIG. 6. The plot of the external tube surface and condensate profile.

Defining

$$\varepsilon = \frac{R_{h_{avg}}}{R_{w_{avg}}} \tag{35}$$

equation (34) takes the form

$$V_z = \frac{\rho g_z}{4\mu} R^2_{w_{avg}} \left[1 - \left(\frac{r}{R_{w_{avg}}} \right)^2 + 2\varepsilon^2 \ln \frac{r}{R_{w_{avg}}} \right]. \tag{36}$$

For the interval $\Delta\gamma$, the volumetric flow rate of the condensate film is defined by

$$\Delta \dot{V} = \int_{\gamma_2}^{\gamma_1} \int_{R_{w_{avg}}}^{R_{h_{avg}}} V_z r \, dr \, d\gamma. \tag{37}$$

After changing the integration variables equation (37) takes the form

$$\Delta \dot{V} = R^2_{w_{avg}} \int_{\gamma_2}^{\gamma_1} \int_1^{\varepsilon} V_z \xi \, d\xi \, d\gamma \tag{38}$$

where

$$\xi = \frac{r}{R_{w_{avg}}}. \tag{39}$$

Substitution of equation (36) into equation (38) and integrating over the interval $\Delta\gamma$ gives

$$\Delta \dot{V} = \Delta\gamma R^2_{w_{avg}} \frac{\rho g_z}{4\mu} \int_1^{\varepsilon} (1 - \xi^2 + 2\xi^2 \ln \xi) \xi \, d\xi. \tag{40}$$

Integrating once more and rearranging yields

$$\Delta \dot{V} = \Delta\gamma R^4_{w_{avg}} \frac{\rho g_z}{16\mu} (4\varepsilon^2 - 3\varepsilon^4 + 4\varepsilon^4 \ln \varepsilon - 1). \tag{41}$$

The overall volumetric flow rate is the summation of the individual $\Delta\dot{V}$s for the given angle γ. Since γ is a function of the arc length, s, the integration limit can be defined by this variable.

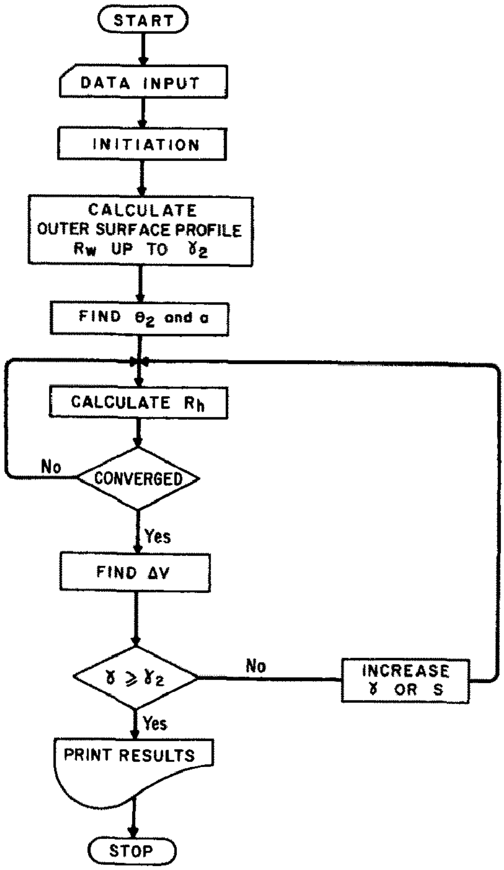


FIG. 7. Algorithm to calculate the condensate film flow rate for a sinusoidal flute profile.

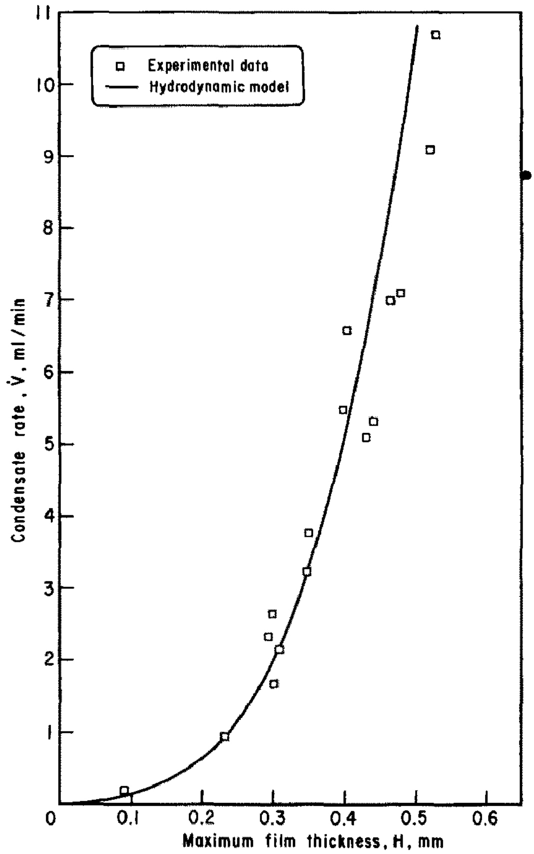


FIG. 8. Relation between condensate rate and maximum film thickness.

The algorithm of the explained model is given in Fig. 7.

The condensate surface profile, R_h, is calculated by using the Wegstein convergence method. The value of R_h calculated in each step in the s-direction is taken as an initial estimate for the next step.

The procedure explained above is repeated for different γ_2 values and a plot indicating the relation between condensate rate and maximum film thickness is prepared. Figure 8 shows how the hydrodynamic model fits the experimental data points obtained by Özgen [12]. It can be seen that the results obtained by our hydrodynamic model are in very good agreement with the experimental results.

5. CONCLUSION

The model used can be applied to other flute geometries and further can be used for heat transfer studies.

REFERENCES

1. R. Gregorig, Hautkondensation auf feingewellten Oberflächen bei Berücksichtigung der Oberflächenspannungen, *Z. Angew. Math. Phys.* **5**, 36–49 (1954).
2. D. G. Thomas, Enhancement of film condensation rate on vertical tubes by longitudinal fins, *A.I.Ch.E. Jl* **14**, 644–649 (1968).
3. L. G. Alexander and H. W. Hoffman, Performance characteristics of corrugated tube for vertical evaporators, ASME Publication 71-HT-30, ASME–AIChE Heat Transfer Conference, Tulsa, Oklahoma, 15–18 August (1971).
4. D. G. Thomas, Prospects for further improvement in enhanced heat transfer surface, *Desalination* **12**, 189–215 (1973).
5. S. K. Combs, Experimental study of ammonia condensation on vertical fluted tubes, 14th Southeastern Seminar on Thermal Sciences, Raleigh, 6–7 April, pp. 65–85 (1978).
6. H. Suematsu, S. Tnoue and Y. Soejima, Condensation heat transfer on fluted tube, *Proceedings of Kagaku Kogaku 8th Shuki Taikai*, pp. 402–403 (1974).
7. T. G. Somer and C. Özgen, Hydrodynamics of condensate films on fluted tube surfaces. Part I, *Desalination* **34**, 233–247 (1980).
8. C. Zener and A. Lavi, Drainage systems for condensation, *Trans. Am. Soc. Mech. Engrs*, Series A, *Engng Pwr* **96**, 209–215 (1974).
9. R. L. Webb, A generalized procedure for the design and optimization of fluted Gregorig condensing surfaces, *Trans. Am. Soc. Mech. Engrs*, Series C, *J. Heat Transfer* **101**, 335–339 (1979).
10. Y. Mori, K. Hijikata, S. Hirasawa and W. Nakayama, Optimized performance of condenser with outside condensing surface, 18th National Heat Transfer Conference, San Diego, California, 6–8 August, pp. 52–62 (1979).
11. V. G. Rifert, A new method for calculating rates of condensation on finned tubes, *Heat Transfer—Sov. Res.* **12**(3) (1980).

12. C. Özgen, Effect of tube surface geometry on the rate of heat transfer in desalination by evaporation, Ph.D. Thesis, Middle East Technical University, Ankara (1978).
13. C. Y. Wang, Diffusive transport across corrugated plates and tubes, *A.I.Ch.E. Jl* **28**, 1016–1019 (1982).
14. R. B. Bird, W. E. Stewart and E. N. Lightfoot, *Transport Phenomena*. Wiley, New York (1960).

UN MODELE HYDRODYNAMIQUE DEVELOPPE POUR LE CONDENSAT QUI S'ECOULE SUR UN TUBE CANNELE SINUSOIDALEMENT

Résumé—Les tubes cannelés conçus par Gregorig en considération de la tension interfaciale sont communément utilisés dans les évaporateurs et les condenseurs pour désalement. Dans cette étude, le profil de surface du tube qui ne peut pas être décrit par des systèmes de coordonnées conventionnels, est obtenu numériquement en utilisant un système intrinsèque. Les équations de Navier–Stokes sont résolues après simplification, pour l'écoulement de condensat sur un profil sinusoïdal, afin de calculer le débit. Les résultats sont trouvés être en bon accord avec les données expérimentales.

EIN HYDRODYNAMISCHES MODELL DER KONDENSATSTRÖMUNG AN EINEM SINUSFÖRMIG PROFILIERTEN ROHR

Zusammenfassung—Profilierte Rohre, wie sie von Gregorig unter Ausnutzung der Oberflächenspannung vorgeschlagen wurden, finden Anwendung in Verdampfern und Kondensatoren bei der Wasserentsalzung. In der vorliegenden Untersuchung wurde das Oberflächenprofil eines Rohres, welches mit keinem herkömmlichen Koordinatensystem beschrieben werden kann, numerisch in einem speziellen Koordinatensystem ermittelt. Die Navier–Stokes-Gleichungen werden nach einigen Vereinfachungen für die Kondensatströmung an einem sinusförmigen Profil gelöst; der Volumenstrom wird ermittelt. Die Ergebnisse stimmen gut mit experimentellen Befunden überein.

ГИДРОДИНАМИЧЕСКАЯ МОДЕЛЬ ТЕЧЕНИЯ КОНДЕНСАТА В СИНУСОИДАЛЬНОЙ ТРУБЕ С ГОФРИРОВАННЫМИ СТЕНКАМИ

Аннотация—Трубы с гофрированными стенками, разработанные Грегоригом с учетом сил натяжения, обычно используются в испарителях и конденсаторах для опреснения воды. В настоящем исследовании с помощью системы собственных координат рассчитан профиль поверхности гофрированной трубы, который нельзя описать с помощью обычной системы координат. С учетом некоторых упрощений решены уравнения Навье–Стокса для потока конденсата в трубе синусоидального профиля, что позволило рассчитать объемную скорость потока. Полученные результаты хорошо совпадают с экспериментальными данными.